Black Hole Thermodynamics in Hor̆ava Lifshitz Gravity and the Related Geometry

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Recently, Hor̆ava proposed a non-relativistic renormalizable theory of gravity which is essentially a field theoretic model for a UV complete theory of gravity and reduces to Einstein gravity with a non-vanishing cosmological constant in IR. Also the theory admits a Lifshitz scale-invariance in time and space with broken Lorentz symmetry at short scale. On the other hand, at large distances higher derivative terms do not contribute and the theory coincides with general relativity. Subsequently, Cai and his collaborators and then Catiuo et al have obtained black hole solutions in this gravity theory and studied the thermodynamic properties of the black hole solution. In the present paper, we have investigated the black hole thermodynamic for two choices of the entropy function - a classical and a topological in nature. Finally, it is examined whether a phase transition is possible or not.

Keywords : Thermodynamics, black hole, phase transition.

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I. INTRODUCTION

Recently, a new four dimensional gravity theory has been proposed by Horava (2009) without full diffeomorphism invariance but UV completeness. In fact, the theory has three dimensional general covariance and time re-parameterization invariance. Essentially, it is a non-relativistic renormalizable quantum gravity theory with higher spatial derivatives. Due to anisotropic rescaling Horava's theory is power-counting renormalizable. At large distance the general covariance is recovered and the theory reduces (with some restrictions) to Einstein gravity with a non-vanishing cosmological constant in IR , i.e., the general IR vacuum of this theory is anti-deSitter. However, if one adds a new term ($\mu^4 R$)in the action and take $\Lambda \to 0$ then although UV properties of the theory do not change but IR properties alters and one gets Minkowskin vacuum in the IR region. Moreover, due to Lifshitz scale invariance ($t \to l^z t, x^i \to l x^i, z \geq 1$) of the space time this theory is also known as Horava-Lifshitz theory (HL). But in semiclassical treatment of scalar excitation of HL gravity shows the usual naturalness problem of Lorentz violating theories.

Horava has used ADM formalism where the four dimensional metric is parameterized as (Arnowitt et. al. 1962)

$$ds^2 = -N^2 dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) \tag{1}$$

Here the lapse function N, shift vector N^i and the 3-Dimensional spatial metric g_{ij} are the dynamical variables in Horava-Lifshitz gravity. So the Eintein-Hilbert action has the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{g}\; N\left\{\left(K_{ij}K^{ij} - K^2\right) + R - 2\Lambda\right\} \tag{2}$$

where

$$G = \; Newton's \; gravitational \; constant,$$

R is the curvature scalar for the 3-metric g_{ij} and the extrinsic curvature K_{ij} is defined as

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j \nabla_j N_i\right) \tag{3}$$

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Here an over dot stands for time derivative while ∇_i is the covariant derivative with respect to the spatial three metric g_{ij}.

The action of the non-relativistic renormalizable theory proposed by Hořava (2009) (known as Hořava-Lifshitz (HL) action) has the expression

$$I = \int dt\ d^3x \sqrt{g}\ N \left(L_0 + \ L_1\right) \tag{4}$$

with

$$L_0 = \left\{ \frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) + \frac{\kappa^2\mu^2\left(\Lambda R - 3\Lambda^2\right)}{8\left(1-3\lambda\right)} \right\}$$

$$L_1 = \left\{ \frac{\kappa^2\mu^2\left((1-4\lambda\right)}{32\left(1-3\lambda\right)}R^2 - \frac{\kappa^2}{2\omega^4}Z_{ij}Z^{ij} \right\}$$

Here

$$Z_{ij} = C_{ij} - \frac{\mu\ \omega^2}{2}R_{ij}\ \ , \tag{5}$$

the Cotten tensor C^{ij} has the expression

$$C^{ij} = \epsilon^{ikl}\nabla_k\left(R^j_l - \frac{1}{4}R\ \delta^j_l\right) = \epsilon^{ikl}\nabla_k R^j_l - \frac{1}{4}\epsilon^{ikj}\partial_k R \tag{6}$$

and $\kappa^2,\ \lambda,\ \mu,\ \omega$ and Λ are constant parameters.

In the above Hořava-Lifshitz action (4) the first two terms are the kinetic terms and the rest correspond to potential of the theory (in $'detailed-balance'$ form). Now comparing with general relativity action, the expressions for the speed of light, Newton's constant and the cosmological constant are

$$c = \frac{\kappa^2\mu}{4}\sqrt{\frac{\Lambda}{1-3\lambda}}\ ,\ G = \frac{\kappa^2 c}{32\pi}\ ,\ \tilde{\Lambda} = \frac{3}{2}\Lambda\ . \tag{7}$$

Note that in the present theory (Horava 2009) λ is a dynamical coupling constant, subject to quantum correction. In fact, for $\lambda = 1$ the first three terms in the action (4) can be converted to the usual ones in Einstein's general relativity. Also the expression for the velocity of light (in equation (7)) demands the cosmological constant Λ must be negative if $\lambda > \frac{1}{3}$. However, an analytic continuation (Lu et. al. 2009)

$$\mu \rightarrow i\mu, \qquad \omega^2 \rightarrow -i\omega^2 \tag{8}$$

keeps the action (4) to be real (Calcagni 2009) and we may choose Λ to be positive for $\lambda > \frac{1}{3}$. The cosmological implications of the HL action has been studied by Kiritsis et. al. (2009), Hořava (2009; 2009), Takahasi et. al. (2009), Kluson (2009), Cai et. al. (2009; 2009).

Now variation of the HL action with respect to N, N^i and g_{ij} give the equations of motion

$$\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2\mu^2\left(\Lambda R - 3\Lambda^2\right)}{8\left(1-3\lambda\right)} - \frac{\kappa^2\mu^2\left(1-4\Lambda\right)}{32\left(1-3\lambda\right)}R^2 + \frac{\kappa^2}{2\omega^4}Z_{ij}Z^{ij} = 0 \tag{9}$$

$$\nabla_k\left(K^{kl} - \lambda K g^{kl}\right) = 0 \tag{10}$$

and

$$\frac{2}{\kappa^2}E^{(1)}_{ij} - \frac{2\lambda}{\kappa^2}E^{(2)}_{ij} + \frac{\kappa^2\mu^2\Lambda}{8\left(1-3\lambda\right)}E^{(3)}_{ij} + \frac{\kappa^2\mu^2\left(1-4\lambda\right)}{32\left(1-3\lambda\right)}E^{(4)}_{ij} - \frac{\mu\kappa^2}{4\omega^2}E^{(5)}_{ij} - \frac{\kappa^2}{2\omega^4}E^{(6)}_{ij} = 0 \tag{11}$$

where the tensors $E_{ij}^{(\alpha)}$ $(\alpha = 1, 2, ..., 6)$ are the combination of $K_{ij},\ g_{ij},\ N,\ N_i$ and their covariant derivative with respect to the three dimensional metric and detailed expressions can be found by Arnowitt et. al. (1962).

Immediately, after the proposal of this new theory, Lu et. al. (2009) have obtained spherically symmetric solutions in HL gravity and have shown asymptotically AdS_2 solution for $\lambda = 1$. Subsequently, their solution has been extended to general topological black hole solution by Cai et al (2009) and they have studied extensively the corresponding black hole thermodynamics. Subsequently the HL theory has been modified by introducing a new form $\mu^4 R$ in the action. Then Kehagias and Sfetsos (2009) has obtained asymptotic flat spherically symmetric vacuum black hole solution (known as KS black hole). For spherically symmetric solution considering $N_i = 0$ in (1) we obtain the metric ansatz

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d^2\Omega \tag{12}$$

From this line element evaluating the angular integration, the modified Lifshitz-Horava Lagrangian reduces to

$$\tilde{\mathcal{L}}_1 = \frac{\kappa^2\mu^2 N}{8\,(1-3\lambda)\sqrt{f}}\left\{\frac{\lambda-1}{2}f'^2 + \frac{(2\lambda-1)\,(f-1)^2}{r^2} - \frac{2\lambda\,(f-1)}{r}f' - 2\omega\,(1-f-rf')\right\} \tag{13}$$

with

$$\omega = \frac{8\mu^2\,(3\lambda-1)}{\kappa^2}$$

Now $\lambda = 1$ gives $\omega = \frac{16\mu^2}{\kappa^2}$ and then the field equations obtained from the Lagrangian (13), can be solved to obtain $'f'$ and $'N'$ as (Kehagias et. al. 2009; Catillo et. al. 2009) the values of f and N can be evaluated as (Kehagias et. al. 2009)

$$N^2 = f(r) = 1 + \omega r^2 - \sqrt{r\,(\omega^2 r^3 + 4\omega M)} \tag{14}$$

In the above solution the integration constant $'M'$ is related to the mass of the corresponding black hole. In fact, if r_+be the radius of the event horizon then

$$M = \frac{r_+}{2} + \frac{1}{4\omega r_+} \tag{15}$$

This expression for M is similar to the mass-charge relation for (non-extremal) Reissner-Nordstrom black hole. Here $\frac{1}{\sqrt{\omega}}$ corresponds to charge parameter in Reissner-Nordstrom solution and is taken as a new parameter in the black hole thermodynamics. If $M^2 > \frac{1}{2\omega}$ then there are more than one horizon while there will be degenerate horizon at $r_c = M = \sqrt{\frac{1}{2\omega}}$(extremal one) and no horizon exists for $M^2 < \frac{1}{2\omega}$ and we are left with a naked singularity.

The nice similarity between black hole and thermodynamical system was first introduced by Bekenstein (1973) and Hawking (1975), by relating the surface gravity of the horizon of the black hole to the temperature of the thermodynamical system(known as Hawking temperature) while the area of the horizon is proportional to the entropy of the thermodynamical system,i.e.,

$$T = \frac{\kappa}{2\pi}\quad ,\quad S = \frac{A}{4} \tag{16}$$

and these entropy and temperature are related by the first law of thermodynamics (Bardeen et. al. 1973). But in general the area formula for entropy breaks down for higher derivative gravity theories. Using first law of thermodynamics we can write entropy as

$$S = \int \frac{dM}{T} + S_0.$$

The integration constant $'S_0'$ should be fixed by physical consideration. As the mass of a black hole is a function of the radius of the horizon r_+ so we can write the above integral as

$$S=\int\frac{1}{T}\frac{\partial M}{\partial r_+}dr_+ + S_0$$

Using M from (15) we have

$$S=\pi r_+^2+\frac{2\pi}{\omega}\ln r_+ \tag{17}$$

(choosing the integration constant $S_0 = 0$ to match with Schwarzschild black hole) This is expression of entropy of the HL black hole using Hawking temperature and first law of thermodynamics on the horizon. Also it is the usual Bekenstein entropy with logarithmic correction. This entropy is similar to that for topological black holes obtained by Cai et. al.(2009). In this connection, one may note that in GR logarithmic corrections in entropy is due to thermodynamic fluctuations around thermal equilibrium.Also the logarithmic correction term is expected to be a generic one in any theory of quantum gravity.

Usually, a black hole is characterized by three parameters namely its mass, charge and angular momentum(known as no hair theorem) and the thermodynamical stability of the black hole is determined by the sign of its heat capacity(c_ω). If $c_\omega < 0$ (as Schwarzschild black hole), then black hole is thermodynamically unstable, but if c_ω changes sign in the parameter space such that it diverges (Hut 1977) in between then from ordinary thermodynamics it indicates a second order phase transition (Davies 1977; 1977; 1989). However in black hole thermodynamics, a critical point exits also for extremal black hole and the second order phase transition takes place from an extremal black hole to its non extremal counter part.

On the other hand the geometrical concept into ordinary thermodynamics was first introduced by Weinhold (1975) (for details see the review (Ruppeiner 1995; 1996)). According to him a Riemannian metric can be defined as the second derivative of the internal energy (U) to entropy (S) and other extensive variables (y^α) of the system ,i.e., the Hessian of the energy is known as Weinhold metric

$$g_{ij}^{(W)}=\partial_i\partial_j U(S,y^\alpha) \tag{18}$$

However it looses physical meaning in equilibrium thermodynamics. Subsequently, Ruppeiner (1979) introduced a metric (known as the Ruppeiner metric) as the Hessian matrix of the thermodynamic entropy. It is defined on the state space as

$$g_{ij}^{(R)}=-\partial_i\partial_j S(U,y^\alpha) \tag{19}$$

The Ruppeiner metric and the Weinhold metric are conformally related as

$$ds_R^2=\frac{1}{T}ds_W^2 \tag{20}$$

Unlike Weinhold metric, the Ruppeiner geometry has physical relevance in the fluctuation theory of equilibrium thermodynamics.

In the present work, we make a comparative study of the black hole thermodynamics for the above modified Horava-Lifshitz gravity with logarithmically corrected entropy (called as entropy-I) given by equation (17) and the usual area-entropy relation (called as entropy-II) given in equation (16). The paper is organized as follows : In section II black hole thermodynamics has been studied for entropy-I and the variation of the thermodynamical quantities has been examined. A similar analysis with entropy-II has been presented in section III.The paper ends with a conclusion in section-IV.

II. BLACK HOLE THERMODYNAMICS IN HORAVA-LIFSHITZ GRAVITY WITH ENTROPY-I

A. Calculations

If r_+ denotes the event horizon then from equation (14) the mass, entropy are given by equation (15) and (17) and other thermodynamical quantities namely temperature, specific heat and free energy are given by,

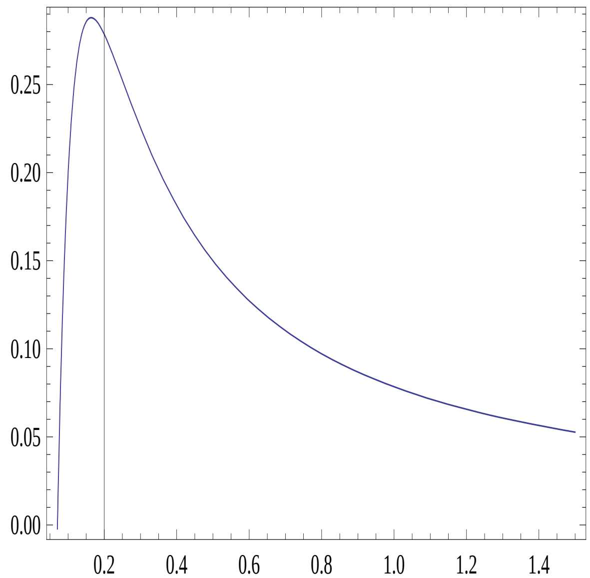

Fig.(1a) T_H vs r_+ graph, ω = 100.

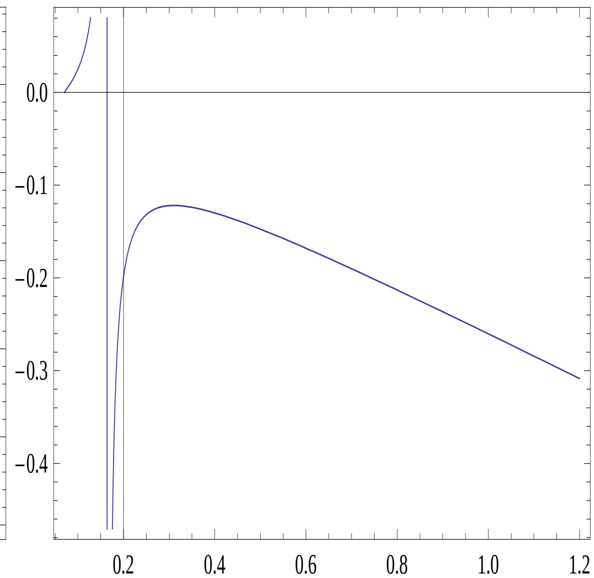

Fig.(1b) c_ω vs r_+ graph, ω =100

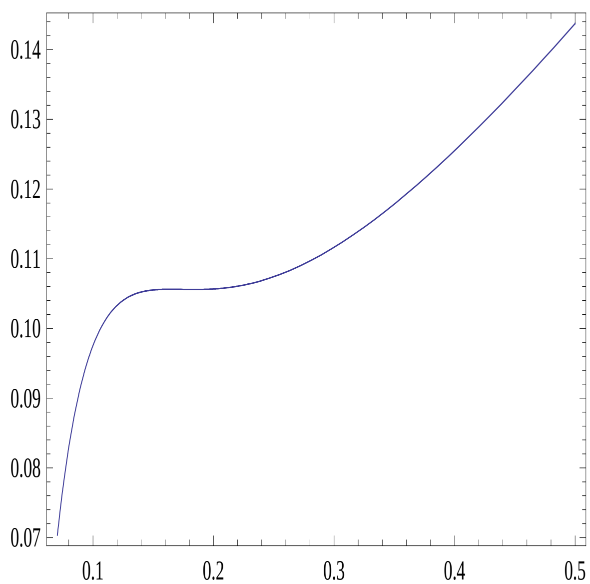

Fig.(1c) F vs r_+ graph, ω = 100.

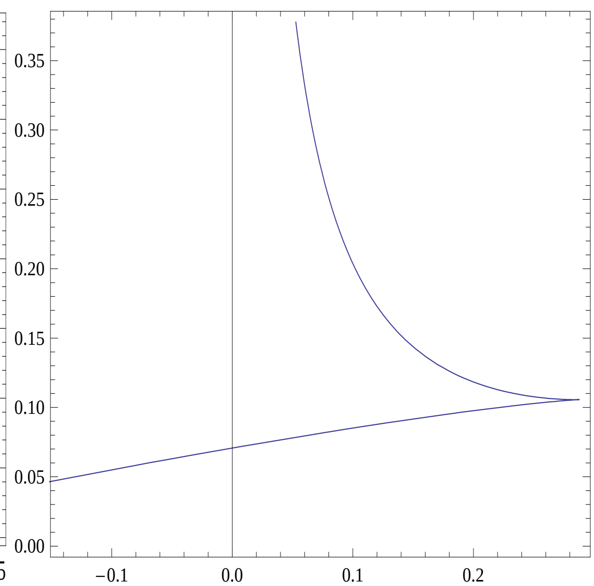

Fig.(1d) F vs T_H graph. ω = 100.

Fig. 1(a), 1(b), 1(c) show the variation of T_H, c_ω and F respectively with r_+. Fig. 1(d) represents the variation of F with respect to T_H

- $$T = \left(\frac{\partial M}{\partial S}\right)_\omega = \frac{2r_+^2\omega - 1}{8\pi r_+\left(1+\omega r_+^2\right)} \tag{21}$$

- $$c_\omega = T\left(\frac{\partial S}{\partial T}\right)_\omega = \frac{2\pi}{\omega}\frac{\left(1+\omega r_+^2\right)^2\left(2\omega r_+^2 - 1\right)}{\left(1+5\omega r_+^2 - 2\omega^2 r_+^4\right)} \tag{22}$$

- $$Free \quad energy \quad (F) = M - S.T = \frac{2 + r_+^2\omega\left(7 + 2r_+^2\omega\right) + 2\left(1 - 2r_+^2\omega\right)\ln r_+}{8r_+\omega\left(1 + r_+^2\omega\right)} \tag{23}$$

Now to derive the Weinhold metric we choose $'\omega'$ as the other extensive variable of the thermodynamic system. The reason for choosing $'\omega'$ as a parameter is that it corresponds to charge in Reissner-Nordstrom solution (mentioned in the introduction). Also recently, Wang et al (2009) have treated $'\omega'$ as a new state parameter in studying the first law of thermodynamics in IR modified HL space time. Thus Weinhold metric has the form,

$$dS_W^2 = \frac{\omega}{16\pi^2 r_+}\frac{\left(1+5r_+^2\omega-2r_+^4\omega^2\right)}{\left(1+r_+^2\omega\right)^3}\ dS^2 + \frac{3}{4\pi}\frac{r_+}{\left(1+r_+\omega\right)^2}\ dSd\omega + \frac{1}{2\omega^3 r_+}\ d\omega^2 \tag{24}$$

and the Ruppeiner metric is

$$dS_R^2 = \frac{1}{T}dS_W^2 = -\frac{\omega}{2\pi}\frac{\left(1+5r_+^2\omega-2r_+^4\omega^2\right)}{\left(1-3r_+^4\omega^2-2r_+^6\omega^3\right)}\ dS^2 + \frac{6r_+^2}{\left(2r_+^4\omega^2+r_+^2\omega-1\right)}\ dSd\omega + \frac{4\pi}{\omega^3}\frac{\left(1+\omega r_+^2\right)}{\left(2\omega r_+^2-1\right)}d\omega^2 \tag{25}$$

and after diagonalizing we get,

$$dS_R^2 = -\frac{\omega}{4\pi}\frac{\left(2+12\omega r_+^2-3\omega^2 r_+^4-4\omega^3 r_+^6\right)}{\left(1-2r_+^2\omega\right)\left(1+r_+^2\omega\right)^3}dS^2 + d\sigma^2 \tag{26}$$

where σ can be determined from the equation

$$d\sigma = \sqrt{\frac{\pi}{\omega^3}\frac{\left(2\omega r_+^2-1\right)}{\left(1+\omega r_+^2\right)}}\left[\frac{\left\{2+\omega r_+^2\left(4-3\ln r_+\right)+2\omega^2 r_+^4\right\}}{\left(2\omega^2 r_+^4+\omega r_+^2-1\right)}d\omega + \frac{3\omega^2 r_+}{\left(2\omega r_+^2-1\right)}dr_+\right] \tag{27}$$

B. Graphical Analysis and Physical Interpretations

We shall now analyze the thermodynamical quantities graphically. The Hawking temperature (T) is zero at $r_+ = r_{+1}$(say) and then increases sharply to a maximum T_M at $r_+ = r_{+2}$ (say). Then T decreases gradually and again approaches to zero asymptotically. Thus a tiny black hole will have zero Hawking temperature, i.e., vanishing surface gravity. As the size of the event horizon increases, the Hawking temperature as well as the surface gravity increases and reaches a maximum value at the event horizon radius r_{+2} Then further increase of the radius of the event horizon results a decrease of Hawking temperature (and surface gravity) and finally infinite black hole will have zero temperature and vanishing surface gravity.

The figure 1(b) shows that c_ω has two distinct branches. Starting from zero, c_ω remains positive for $r_{+1} < r_+ < r_{+2}$. At $r = r_{+2}$, c_ω blows up and changes sign. For $r_+ > r_{+2}$, c_ω remains negative. From negative infinity value c_ω increases sharply reaches a maximum and then gradually decreases.

The graph for F plotted against r_+ (in figure (1c)) shows that free energy is positive except for small $r_+(< r_{+1})$. The curve has a point of inflexion initially concave downwards and then concave upwards.

Also the $F-T$ variation in figure (1d) shows that there are two branches of the curve. For one of the branches the free energy increases very slowly with the increase of the Hawking temperature to the maximum limit T_M while for the other branch F starts from infinite value decreases sharply till $T = T_M$. Here $T = T_M$ is a cusp type double point.

Finally, we note that $r_{+1} = \frac{1}{\sqrt{2\omega}}$ is the minimum possible radius of the event horizon. At $r_+ = r_{+2}$(the positive real root of $1+5r_+^2\omega-2r_+^4\omega^2 = 0$) there is a possible phase transition. Here c_ω blows up and changes sign (from positive to negative), the Ruppeiner metric becomes degenerate and in $F-T$ diagram it corresponds to a cusp type double point. In analogy with second order Hawking-Page (1983) type phase transition (which occurs in the AdS or asymptotically AdS black hole) the black hole in HL gravity (which asymptotically behaves like Minkowskian one) admits a transition from initial stable phase to an unstable one.

Further, at $r_+ = r_{+1} = \frac{1}{\sqrt{2\omega}}$, the vanishing of the Hawking temperature indicates the equality of the free energy with the mass of the black hole and we have $F = M = r_+ = \frac{1}{2\omega}$.

III. BLACK HOLE THERMODYNAMICS IN HORAVA-LIFSHITZ GRAVITY WITH ENTROPY-II

This section analyze the thermodynamical parameters with entropy-II. Although, the entropy area relation holds in GR but still it is interesting to study the black hole thermodynamics with entropy-II in HL gravity theory for a comparative study as HL gravity corresponds to (with some restrictions) to GR in IR cut off. In this section, one may note that recently Wang et al (2009) have studied the first law of thermodynamics in IR modified HL-space time using entropy-II.

A. Calculations

From equation (14) we have horizons at

- $$r_{\pm} = M \pm \sqrt{M^2 - \frac{1}{2\omega}} \qquad (28)$$

The surface area of event horizon is given by

$$A = r_+^2 \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin\theta \; d\theta \; d\phi = 4\pi r_+^2$$

So entropy of the the black holes from equation (16) has the form

- $$S = \pi r_+^2 \qquad (29)$$

and from equations (28) and (29) we have,

- $$M = \frac{1}{2\sqrt{\pi}} \left[S^{\frac{1}{2}} + \frac{\pi}{2\omega} S^{-\frac{1}{2}} \right] \qquad (30)$$

- $$T = \left(\frac{\partial M}{\partial S} \right)_{\omega} = \frac{2\omega S - \pi}{8\sqrt{\pi}\omega S^{\frac{3}{2}}} \qquad (31)$$

- $$c_{\omega} = T \left(\frac{\partial S}{\partial T} \right)_{\omega} = \frac{2S\left(2S\omega - \pi\right)}{\left(3\pi - 2S\omega\right)} \qquad (32)$$

- $$Free \;\; energy \;\; (F) = M - T.S = \frac{1}{8\omega\sqrt{\pi}S^{\frac{1}{2}}} \left(3\pi + 2\omega S\right) \qquad (33)$$

In such case Weinhold metric is as follows

$$dS_W^2 = \frac{(3\pi - 2S\omega)}{16\sqrt{\pi}S^{\frac{5}{2}}\omega} dS^2 + \frac{\sqrt{\pi} dS \; d\omega}{4S^{\frac{3}{2}}\omega^2} + \frac{\sqrt{\pi} d\omega^2}{2S^{\frac{1}{2}}\omega^3} \qquad (34)$$

Ruppeiner metric has the form,

$$dS_R^2 = \frac{2S\omega - 3\pi}{2S\left(\pi - 2\omega S\right)} dS^2 + \frac{2\pi dS \; d\omega}{\omega\left(2S\omega - \pi\right)} + \frac{4\pi S d\omega^2}{\omega^2\left(2S\omega - \pi\right)}, \qquad (35)$$

which after diagonalization becomes

$$dS_R^2 = \frac{\pi S - 3\pi\omega^2 + 2S\omega^3}{2S\left(\pi\omega^2 - \pi S - 2S\omega^3\right)} dS^2 + d\sigma^2. \qquad (36)$$

In this case σcan be determined by the equation

$$d\sigma = \frac{1}{2\omega} \frac{\pi^{\frac{1}{2}}}{S^{\frac{1}{2}}\left(2S\omega - \pi\right)^{\frac{1}{2}}} \left\{ 8S \; d\omega \; + \omega \; dS \right\} \qquad (37)$$

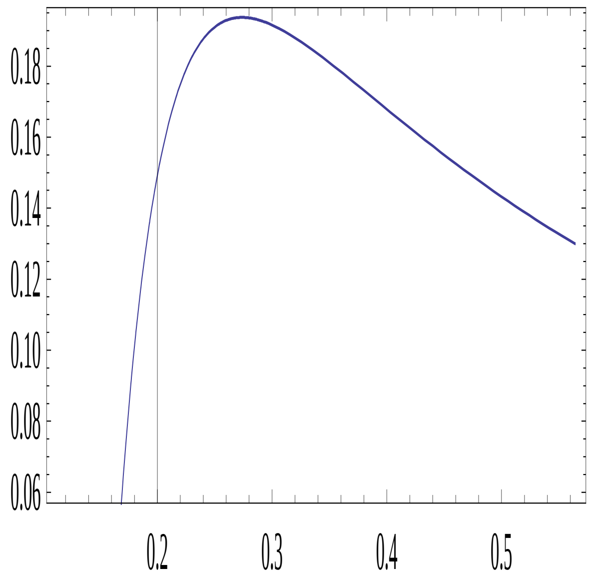

Fig.(2a) T vs r_+ graph for $\omega = 20$.

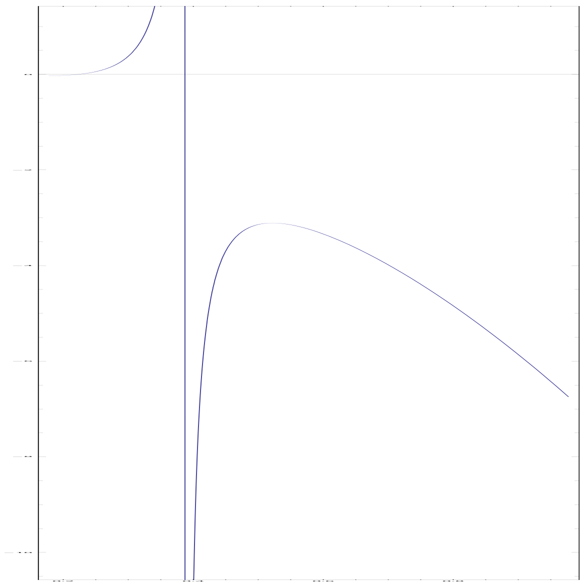

Fig.(2b)c_ω vs r_+ graph for $\omega = 10$

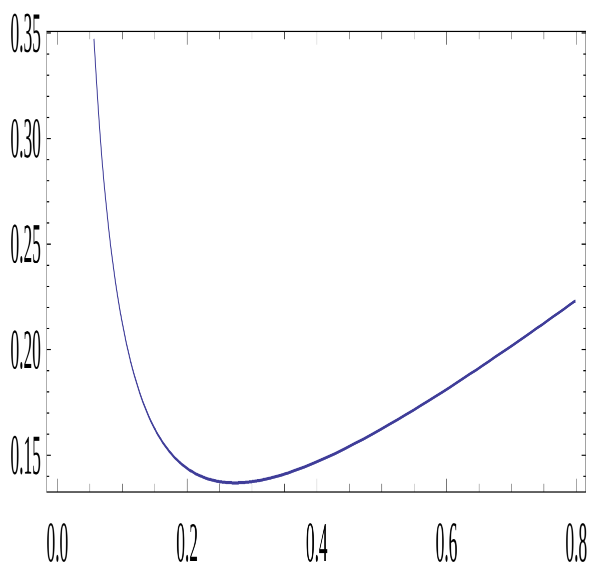

Fig.(2c) F vs r_+ graph for ω =20

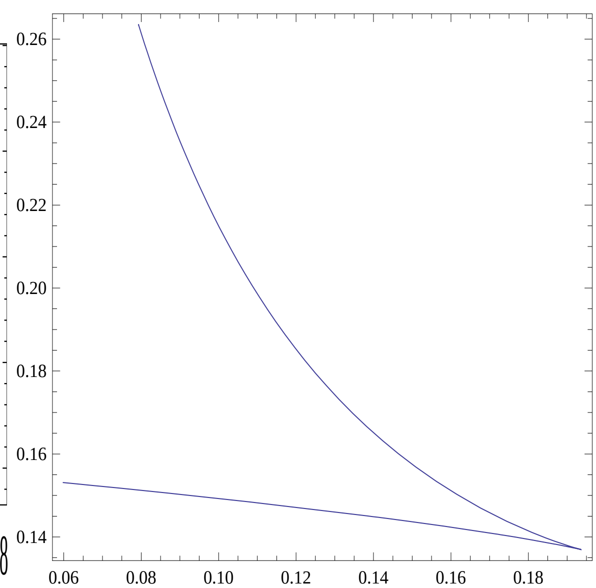

Fig.(2e) F vs T graph, $\omega = 20$.

Fig. 2(a), 2(b) and 2(c) show the variation of T, c_ω and F respectively with r_+. Fig 2(d)represents variation of F with T

B. Graphical Analysis and Physical Interpretations

The graphical representation of the thermodynamical quantities in figures ($2a$)-($2d$) show very similar behavior as in previous case. The figures for T and c_ω are identical to the earlier ones. Here also the minimum value of the radius of the event horizon is $r_{+1} = \frac{1}{\sqrt{2\omega}}$ at which both T and c_ω are zero. T has the maximum value ($=T_M^*$) at $r_+ = \sqrt{\frac{3}{2\omega}}$ where c_ω diverges and changes sign.

The graph of F against r_+ in figure ($2c$) is distinct from that in the last section. Although, in both cases F is positive throughout but previously F is always an increasing function with a point of inflexion while in the present case F starts from an extreme large value, decreases sharply till a minima and then increases gradually.

The variation of F against T is shown in figure ($2d$). As in the previous case F has two branches with

a cusp at $T = T_M^*$ (*i.e.*, $r_+ = \sqrt{\frac{3}{2\omega}}$). Hence in this case there is also a phase transition at $r_+ = \sqrt{\frac{3}{2\omega}}$ of Hawking-Page type (with degenerate Ruppeiner metric) and the black hole finally becomes an unstable one.

IV. CONCLUSION

We have studied the thermodynamics of the black holes in modified Hořava Lifshitz gravity (known as KS black hole) for two choices of entropy - one classical and the other is a topological one. For both choices, the black hole is initially stable and becomes unstable through a Hawking page type phase transition. The free energy F is positive in both the cases supportingly the unstable nature of the black hole. However, the physical significance of the point of inflexion for the curve of F in figure $1(c)$ is not clear. The maximum temperature representing the cusp type double point in the $F - T$ curve corresponds to phase transition - whether a general feature or not - is not known. Although, the thermodynamics of this KS- black hole has similarity with RN black hole but the thermodynamic geometries are quite distinct. The KS black hole has non-vanishing curvature indicating the statistical system to be interacting. Finally, we conclude that the black hole thermodynamics in modified Hořava Lifshitz gravity for the above two choices of the entropy function is more or less identical. Lastly, the large black hole in modified Hořava Lifshitz gravity is unstable globally while in Einstein gravity Schwartzschild AdS black hole is a stable one.

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